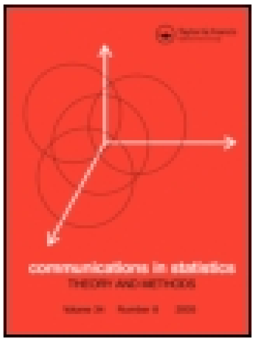

Communications in Statistics - Theory and Methods

ISSN: 0361-0926 (Print) 1532-415X (Online) Journal homepage: http://www.tandfonline.com/loi/lsta20

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To cite this article: Zhiyong Zhang, Kaifeng Jiang, Haiyan Liu & In-Sue Oh (2017): Bayesian meta-analysis of correlation coefficients through power prior, Communications in Statistics - Theory and Methods, DOI: 10.1080/03610926.2017.1288251

To link to this article: http://dx.doi.org/10.1080/03610926.2017.1288251

Accepted author version posted online: 10 Feb 2017.

BAYESIAN META-ANALYSIS OF CORRELATION COEFFICIENTS THROUGH POWER PRIOR

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Abstract

This paper proposes a Bayesian approach for meta-analysis of correlation coefficients through power prior. The primary purpose of this method is to allow meta-analytic researchers to evaluate the contribution and influence of each individual study to the estimated overall effect size though power prior. We use the relationship between high-performance work systems and financial performance as an example to illustrate how to apply this method. We also introduce free online software that can be used to conduct Bayesian meta-analysis proposed in this study. Implications and future directions are also discussed in this article.

Keywords: Meta analysis, correlation coefficient, Bayesian, Power prior

1 INTRODUCTION

Meta-analysis is a statistical method of synthesizing findings from multiple studies to get a more comprehensive understanding of the population [Hunter and Schmidt, 2004]. A simple way to synthesize studies is to calculate the weighted average of correlations between two variables (or differences between two treatments) with a function of the sample size being the weight [e.g., Hunter and Schmidt, 2004]. Both fixed-effects and random-effects models have been used in meta-analysis [e.g., Field, 2001; Hedges and Olkin, 1985; Hedges and Vevea, 1998; Hunter and Schmidt, 2004]. Fixed-effects models assume the true effects are the same and the finding from each study provides an estimate, ideally unbiased or consistent, of it. Random-effects models allow the true effects to be different and heterogeneous and can estimate the between-study variance of the effects. The general consensus is that the random-effects models should always be used because the fixed-effects models can be viewed as special cases of them [e.g., Schmidt et al., 2009; Schmidt and Raju, 2007].

Meta-analysis has been conducted within both the frequentist and Bayesian frameworks although arguably meta-analysis can naturally be viewed as a Bayesian method in general. The frequentist methods for meta-analysis can be found in many places such as Hedges and Olkin [1985], Hunter and Schmidt [2004], and Rosenthal [1991]. There are also studies that have discussed Bayesian meta-analysis [e.g., Brannick, 2001; Carlin, 1992; Morris and Normand, 1992; Schmidt and Hunter, 1977; Schmidt and Raju, 2007; Smith et al., 1995; Steel and Kammeyer-Mueller, 2008], which has been considered as having several advantages, such as "full allowance for all parameter uncertainty in the model, the ability to include other pertinent information that would otherwise be excluded, and the ability to extend the models to accommodate more complex, but frequently occurring, scenarios" [Sutton and Abrams, 2001, p. 277].

Traditional meta-analysis, using either the frequentist or Bayesian approach, typically treats each study of the same quality. Therefore, each study contributes equally to the estimated effect size after controlling the sample sizes (Ns). This makes sense in the areas such as medical research where there is relatively less noise in the data and, therefore, N is often everything. However, in social and behavioral research, not all studies included in a meta-analysis should make equal contribution to the estimated effect size; treating them equivalently might cause unexpected consequences in meta-analysis. For example, strategic management scholars may be interested in the relationships between financial performance and its antecedents, such as human resource management (HRM) practices [Combs et al., 2006] and human capital [Crook et al., 2011]. Financial performance can be measured objectively using data from archival data or subjectively using survey data. Although both objective and subjective measures are widely adopted in the literature, objective information may reflect a firm's financial status more accurately than subjective ratings because the latter involves more cognitively demanding assessments and the informants may not always have the best knowledge of the information. Therefore, those using objective measures may provide more reliable information of the relationships between financial performance and other variables than those based on subjective measures. For another example, due to the difficulty of collecting longitudinal data, longitudinal studies often result in a relatively smaller sample size compared with cross-sectional studies obtaining all information from a single source. Even though longitudinal designs may help avoid common method problems and reduce inflation of correlations [Podsakoff et al., 2003], their small sample sizes make them contribute less to the final result. Instead, the cross-sectional studies with inflated relationships may easily dominate the overall effect size because of their large sample sizes. As illustrated in the two examples, treating individual studies equivalently may produce potential misleading results. Therefore, it is extremely important to understand the effect of each study to the overall effect size in meta-analysis.

In this paper, we propose to evaluate the contribution of a study through power prior. Especially, we focus on the meta-analysis of sample correlation although the same method can be applied to other effect size measures. In the following, we first demonstrate the use of power prior through a fixed-effects model and then we extend our method to random-effect models and meta-regression. Free online software is introduced to carry out the Bayesian meta-analysis discussed in this study. The use of Bayesian meta-analysis is further demonstrated through a real meta-analysis example.

2 BAYESIAN META-ANALYSIS THROUGH POWER PRIOR

The proposed method is derived based on the Fisher z-transformation of correlation. Suppose ρ is the population correlation of two variables that follow a bivariate normal distribution. For a given sample correlation r from a sample of n independent subjects, its Fisher z-transformation, denoted by z, is defined as

$$z = \frac{1}{2} \ln \frac{1+r}{1-r}.$$

z approximately follows a normal distribution with mean

$$\frac{1}{2} \ln \frac{1+\rho}{1-\rho}$$

and variance $\phi = \frac{1}{n-3}$ [Fisher et al., 1921].

Meta-analysis of correlation concerns the analysis of correlation between two variables when a set of studies regarding the relationship between the two variables are available. Suppose there are m studies that report the sample correlation between two variables. Each study reports a sample correlation r_i with the corresponding sample size n_i. Let $z_i = \frac{1}{2} \ln \frac{1+r_i}{1-r_i}$ denote the Fisher z-transformation of r_i and $\zeta_i = \frac{1}{2} \ln \frac{1+\rho_i}{1-\rho_i}$ be the Fisher z-transformation of the population correlation. Then, $z_i \sim N(\zeta_i, \phi_i)$ with $\phi_i = (n_i - 3)^{-1}$.

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2.1 Fixed-effects Models

We first investigate the situation where the population can be considered as homogeneous and, therefore, a fixed-effects model can be used. In this case, the population correlation is

$$\zeta_i \equiv \zeta = \frac{1}{2} \ln \frac{1+\rho}{1-\rho}$$

and $z_i \sim N(\zeta, \phi_i)$.

The use of Bayesian methods requires the specification of priors [Gelman et al., 2003], which provides a perfect way to conduct meta-analysis. A prior represents information on the population correlation, or its Fisher z-transformation, without any data collection. Although a prior is required, it may consist of "no" information through certain types of prior such as Jeffreys' prior [e.g., Gill, 2002; Jeffreys, 1946]. For the fixed-effects models, we need a prior for ζ. Suppose the prior for ζ follows a normal distribution $N(\zeta_0, \psi_0)$ where ζ_0 and ψ_0 are per-determined values. For example, ζ could have a prior N(0,1), which means a researcher initially believes the mean value of ζ is 0, corresponding with a correlation 0, with variance 1. If little to none information is available, the so-called diffuse prior can be used by specifying a large variance such as $\psi_0 = 10^8$.

After collecting data, in the framework of meta-analysis, with the availability of a study, one can get a better picture about the population correlation. Bayesian methods provide a way to update the information on the population correlation through Bayes' Theorem. Let z_1 denote the new information on the correlation after Fisher z-transformation and $z_1 \sim N(\zeta, \phi_1)$. The distribution of the population correlation ζ by combining the prior and the study is

$$p(\zeta|z_1) = \frac{p(\zeta)p(z_1|\zeta)}{p(z_1)},$$

where $p(\zeta|z_1)$ is called the posterior of ζ after combining the information in z_1. From Appendix A, we can conclude that the posterior distribution is also a normal distribution $N(\zeta_1, \psi_1)$ where

$$\zeta_1 = \frac{\frac{1}{\psi_0}\zeta_0 + \frac{1}{\phi_1}z_1}{\frac{1}{\psi_0} + \frac{1}{\phi_1}} \quad (1)$$

$$\psi_1 = \frac{1}{\frac{1}{\psi_0} + \frac{1}{\phi_1}}. \quad (2)$$

Therefore, the posterior mean ζ_1 is the weighted average of prior mean ζ_0 and z_1, where the weights are the inverse of the variances of prior and data. If the prior is very informative, e.g., with a small variance, the prior mean will exert a big effect on the posterior. For an extreme case, if $\psi_0 = 0$, the posterior mean is ζ_0, which is also the prior mean. On the other hand, if only little prior information is available, reflected by a large variance of the prior, the prior mean has little influence on the posterior. For a special case where $\psi_0 = +\infty$, the posterior mean is z_1, and therefore, the posterior is fully determined by data.

The above analysis assumes that z_1 is fully reliable or the researcher wants to utilize full information from z_1. However, if, for practical reason, the information in z_1 is not accurate enough (e.g., obtained from a flawed research design), it might distort the posterior. In this situation, a researcher might prefer using only partial information from z_1. Using the power prior idea developed by Ibrahim and Chen [2000a], we can get the posterior

$$p(\zeta|z_1, \alpha_1) = \frac{p(\zeta)[p(z_1|\zeta)]^{\alpha_1}}{p(z_1)}, \tag{3}$$

where α_1 is a power parameter. Note that if $\alpha_1 = 0$, no information from z_1 is used whereas when $\alpha_1 = 1$, full information of z_1 is used. Partial information of z_1 can be utilized by setting α_1 to be a value between 0 and 1. It can be shown (see Appendix B) that the posterior is still a normal distribution with $N(\zeta_1^*, \psi_1^*)$ where

$$\zeta_1^* = \frac{\frac{1}{\psi_0}\zeta_0 + \frac{\alpha_1}{\phi_1}z_1}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1}}$$

$$\psi_1^* = \frac{1}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1}}.$$

Again the posterior mean is a weighted average of the prior mean and z_1. However, note that the weight is different from the previous situation because it is related to the power α_1. If $\alpha_1 < 1$, then the weight for z_1 is smaller than the one in Equation 1. This means the posterior will rely more on the prior.

Suppose without data collection, a researcher's prior information on ζ is $N(0, 1)$. One study in the literature reported a correlation 0.5 with the sample size 28 and, therefore, $z_1 = 0.549$

with variance 0.04. Table 1 shows the posterior mean and variance for ζ with power α_1 ranges from 0 to 1. When $\alpha_1 = 0$, the posterior is the same as the prior. When α_1 increases from 0.1 to 1, the posterior mean changes towards to z_1 because more information from z_1 is included in the posterior. Furthermore, the posterior variance is also becoming smaller. In summary, the use of power α_1 influences both the posterior mean and posterior variance and can control the contribution of data to the posterior.

In meta-analysis, data from multiple studies are available. Bayesian methods provide a natural way to combine the data together. For example, suppose we have another study with transformed correlation z_2 and its variance ϕ_2 as well as the sample size n_2. Furthermore, the power α_2 is used when combining this study. We have already obtained the posterior of ζ with the first study in Equation 3. To get the posterior by combining z_2, we can simply view the posterior in Equation 3 as a new prior. Then, the posterior of ζ with both z_1 and z_2 is

$$p(\zeta|z_1, z_2, \alpha_1, \alpha_2) = \frac{p(\zeta|z_1, \alpha_1)[p(z_2|\zeta)]^{\alpha_2}}{p(z_2)}.$$

From Appendix C, the posterior distribution is a normal distribution $N(\zeta_2^*, \psi_2^*)$ where

$$\begin{aligned}\zeta_2^* &= \frac{\frac{1}{\psi_0}\zeta_0 + \frac{\alpha_1}{\phi_1}z_1 + \frac{\alpha_2}{\phi_2}z_2}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1} + \frac{\alpha_2}{\phi_2}} \\ \psi_2^* &= \frac{1}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1} + \frac{\alpha_2}{\phi_2}}.\end{aligned}$$

Clearly, the posterior mean is a weighted average of prior and the two studies. More generally, if we have m studies with z_i, n_i, and α_i, the posterior distribution of ζ is $N(\zeta_m^*, \psi_m^*)$ with

$$\begin{aligned}\zeta_m^* &= \frac{\frac{1}{\psi_0}\zeta_0 + \sum_{i=1}^m \frac{a_i}{\phi_i}z_i}{\frac{1}{\psi_0} + \sum_{i=1}^m \frac{a_i}{\phi_i}} \\ \psi_m^* &= \frac{1}{\frac{1}{\psi_0} + \sum_{i=1}^m \frac{a_i}{\phi_i}}.\end{aligned}$$

For illustration, we show the combination of two studies where the first study reported a correlation 0.5 with the sample size 28 and the second study reported a correlation 0 with the sample size 103. Therefore, $z_1 = 0.549$ with variance 0.04 and $z_2 = 0$ with variance 0.01. A diffuse prior $N(0, 100)$ is used here so that the effect of prior is minimized. Table 2 presents the posterior mean and variance of the population correlation with different combinations of power for the two studies. First, when no information from the two studies is utilized ($\alpha_1 = \alpha_2 = 0$), the posterior is just the prior. Second, when only the information of Study 1 is fully used ($\alpha_1 = 1$, $\alpha_2 = 0$), the posterior mean and variance are essentially the same as the Fisher z-transformation and the variance of Study 1 because of the use of the diffuse prior. Similarly, one can solely use the information from Study 2 by setting $\alpha_1 = 0$ and $\alpha_2 = 1$. Third, when the information of the two studies are used fully ($\alpha_1 = \alpha_2 = 1$), the posterior mean is about 0.110, the weighted average of 0.549 and 0 but leaning towards 0 because the second study has a larger sample size and thus a smaller variance. When setting $\alpha_1 = \alpha_2 = 0.5$, the posterior mean is still 0.110 but the variance is about 0.016, twice of that when $\alpha_1 = \alpha_2 = 1$. This is because only partial information is used from the two studies. Similar results can be seen from the table when other combination of power is used. In summary, by controlling the power parameter, one can control the contribution of each study to meta-analysis.

2.2 Random-effects Models

When the population is not homogeneous, it is not reasonable to assume that z_i has the same mean ζ. Therefore, we discuss the random-effects models in the Bayesian framework. A random-effects model can be written as a two-level model,

$$\begin{cases} z_i = \zeta_i + e_i \\ \zeta_i = \zeta + v_i \end{cases} \tag{4}$$

where $Var(e_i) = \phi_i$ and $Var(v_i) = \tau$. The parameter τ represents the between-study variance. In the model, each z_i has its mean ζ_i and the grand mean of ζ_i is ζ. Based on Fisher z-transformation, $z_i \sim N(\zeta_i, \phi_i)$. It is often assumed that v_i has a normal distribution and, therefore, $\zeta_i \sim N(\zeta, \tau)$. For the random-effects model, we have the fixed-effects parameter ζ and variance parameter τ. The parameter τ represents the between-study variability. The parameter ζ can be transformed back to correlation that represents the overall correlation across all studies. In addition, we can also estimate the random effects ζ_i, which can be transformed back to correlations for individual studies.

As for the fixed-effects models, to estimate model parameters for the random-effects models, we need to specify priors. In this study, the normal prior $N(\zeta_0, \psi_0)$ is used for ζ and the inverse gamma prior $IG(\delta_0, \gamma_0)$ is used for τ with ζ_0, ψ_0, δ_0 and γ_0 denoting known constants. In practice, $\zeta_0 = 0$, $\psi_0 = 10^6$, $\delta_0 = 10^{-3}$ and $\gamma_0 = 10^{-3}$ are often used to reduce the influence of priors. With the priors, the conditional posteriors for ζ, τ, and ζ_i can be obtained as in Appendix D. Then, the following Gibbs sampling procedure can be used to get a Markov chain for each parameter.

Choose a set of initial values for ζ and τ, e.g., $\zeta^{(0)} = 0$ and $\tau^{(0)} = 1$.

Generate $\zeta_i^{(1)}, i = 1, \ldots, m$ from the normal distribution

$$N\left(\frac{\frac{\zeta^{(0)}}{\tau^{(0)}} + \frac{z_i\alpha_i}{\phi_i}}{\frac{1}{\tau^{(0)}} + \frac{\alpha_i}{\phi_i}}, \frac{1}{\frac{1}{\tau^{(0)}} + \frac{\alpha_i}{\phi_i}}\right).$$

Generate $\tau^{(1)}$ from the inverse Gamma distribution $\mathrm{IG}(\delta_0 + m/2, \gamma_0 + [\sum_{i=1}^m (\zeta_i^{(1)} - \zeta^{(0)})^2]/2)$.

Generate $\zeta^{(1)}$ from the normal distribution

$$N\left(\frac{\frac{\sum_{i=1}^m \zeta_i^{(1)}}{\tau^{(1)}} + \frac{\zeta_0}{\psi_0}}{\frac{m}{\tau^{(1)}} + \frac{1}{\psi_0}}, \frac{1}{\frac{m}{\tau^{(1)}} + \frac{1}{\psi_0}}\right).$$

Let $\zeta^{(0)} = \zeta^{(1)}$ and $\tau^{(0)} = \tau^{(1)}$ and repeat Steps 2-4 to get $\zeta^{(2)}$, $\tau^{(2)}$ and $\zeta_i^{(2)}, i = 1, \ldots, m$. The above algorithm can be repeated for R times to get a Markov chain for ζ, τ, and ζ_i. It can be shown that the Markov chains converge to their marginal distributions after a certain period and therefore can be used to infer on the parameters [e.g., Gelman et al., 2003]. The period for the Markov chains to converge is called the burn-in period. Suppose the burn-in period is k. Then the rest of the Markov chain from $(k+1)$th iteration to the Rth iteration can be used to get the mean and variance of ζ, τ, and ζ_i. Because a researcher is ultimately interested in the correlations, we can also get the Markov chains for $\rho = \frac{\exp(2\zeta)-1}{\exp(2\zeta)+1}$ and for $\rho_i = \frac{\exp(2\zeta_i)-1}{\exp(2\zeta_i)+1}$.

To illustrate the influence of power parameters on the random-effects meta-analysis, we consider a simple example with three studies that report correlations 0.5, 0 and -0.5 with sample sizes 103, 28 and 103. The Fisher z-transformed data and their variances are given in Table 3.

Table 3 also reports the estimated overall correlation ρ and individual correlation $\rho_i, i = 1, 2, 3$. When the power coefficients are 1 for all three studies, the estimated ρ is approximately 0. Note that the estimated individual population correlations for the first and third studies are smaller than the observed ones. This is called "shrinkage" or "multilevel averaging" effect of multilevel analysis [e.g., Carlin and Louis, 1996; Greenland, 2000; Strenio et al., 1983]. The estimated random effects are pulled towards the average effect. If, based on expert opinions or other information, we suspect the reported negative correlation could be due to the low quality of the study design, we might assign it a different weight. For example, if we give the third study a power coefficient 0.1, the estimated overall effect becomes 0.061. Furthermore, if we assign a power coefficient 0.01, the overall effect becomes 0.215. Therefore, the effect of the observed negative correlation can be controlled through the chosen power coefficients.

2.3 Meta-regression Models

When a random-effects model is suggested, it often indicates possible between-study heterogeneity. Therefore, predictors or covariates can be identified to explain such a heterogeneity. Suppose a set of p covariates are available, denoted by $x_1, x_2, \ldots, x_p$. Then, a meta-regression model can be constructed as below

$$\begin{cases} z_i = \zeta_i + e_i \\ \zeta_i = \beta_1 + \beta_2 x_{1i} + \cdots + \beta_{p+1} x_{pi} + v_i = \mathbf{x}_i \boldsymbol{\beta} + v_i \end{cases}, \tag{5}$$

where $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_{p+1})'$, $\mathbf{x}_i = (1, x_{1i}, x_{2i}, \ldots, x_{pi})$, and $v_i \sim N(0, \tau)$. If a coefficient β_i is significant, x_p is a significant predictor that might be related to the between-study heterogeneity.

To estimate $\boldsymbol{\beta}$ and ϕ, we specify the multivariate normal prior for $\boldsymbol{\beta}$ as $N(\boldsymbol{\zeta}_0, \boldsymbol{\Psi}_0)$ and the inverse Gamma prior $IG(\delta_0, \gamma_0)$ for τ. Typically, we use the following hyper-parameters for the priors: $\boldsymbol{\zeta}_0 = \mathbf{0}_{(p+1)\times 1}$, $\boldsymbol{\Psi}_0 = 10^6 \mathbf{I}$ with $\mathbf{I}$ denoting a $(p+1) \times (p+1)$ identity matrix, and $\delta_0 = \gamma_0 = 10^{-3}$.

With the prior, the conditional posteriors for $\boldsymbol{\beta}$, τ, and ζ_i can be obtained as shown in Appendix E. The conditional posterior distribution of τ is an inverse Gamma distribution $\tau | \boldsymbol{\beta}, \zeta_i \sim$

$IG(\delta_0 + m/2, \gamma_0 + \sum_{i=1}^{m}(\zeta_i - \mathbf{x}_i\beta)^2/2)$. The conditional posterior distribution for β is still a multivariate normal distribution

$$N\left(\left(\mathbf{\Psi}_0^{-1} + \frac{\mathbf{X}'\mathbf{X}}{\tau}\right)^{-1}\left(\mathbf{\Psi}_0^{-1}\zeta_0 + \frac{\mathbf{X}'\mathbf{X}}{\tau}\hat{\boldsymbol{\beta}}\right), (\mathbf{\Psi}_0^{-1} + \frac{\mathbf{X}'\mathbf{X}}{\tau})^{-1}\right)$$

where $\hat{\boldsymbol{\beta}}$ is the least square estimate of $\boldsymbol{\beta}$ such that $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\zeta}$ with $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_m)'$ as the design matrix and $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \ldots, \zeta_m)'$. The conditional posterior for ζ_i is

$$N\left(\frac{\frac{\alpha_i z_i}{\phi_i} + \frac{X_i\beta}{\tau}}{\frac{\alpha_i}{\phi_i} + \frac{1}{\tau}}, \frac{1}{\frac{\alpha_i}{\phi_i} + \frac{1}{\tau}}\right).$$

With the set of conditional posteriors, the Gibbs sampling algorithm can be used to generate Markov chain for each unknown parameter as for the random-effects meta-analysis.

3 SOFTWARE

To facilitate the use of Bayesian meta-analysis method through power prior, we developed a free online program that can be accessed with the URL `http://webbugs.psychstat.org/modules/metacorr/`. The online program can be used within a typical Web browser. It has an interface shown in Figure 1. To use the program, one needs either to upload a new data file or select an existing file. Note names of the existing files are shown in the drop down menu. The existing file has to be a text file in which the data values are separated by one or more white spaces. The first line of the data file should be the variable names, which will be used in the model.

Next, a user chooses a model to use. For example, the user can choose to use either the random-effects model (default option) or the fixed-effects model. Then, information on the model can be provided. Both the *Correlation* and *Sample size* are required for all analysis, which can be specified using the variable names in the data set. For example, if we use “fi” to represent the correlation between financial performance and another variable in the data set, then “fi” should be input in the field of *Correlation* in the interface. Similarly, “n” is used in the Sample size field because in the data set, “n” is the variable name for sample size. In addition, a user can also specify the variables for power coefficients used in the power prior and covariates used in the model.

Finally, one can choose to control the Markov chain Monte Carlo (MCMC) method and output of the meta-analysis. For example, the total number of Monte Carlo iteration and the burn-in period can be specified. In the output, one can require the output of the estimates for random effects ζ_i, DIC, and diagnostic plots for all model parameters including the random effects. If one checks the option *Email notification*, an email will be sent to the user once the analysis is completed.

4 AN EXAMPLE

We use the relationship between high-performance work systems (HPWS) and financial performance as an example to illustrate the use of Bayesian meta-analysis with power prior. HPWS refers to a bundle of human resource management (HRM) practices that are intended to enhance employees' abilities, motivation, and opportunity to make contribution to organizational effectiveness, including practices such as selective hiring, extensive training, internal promotion, developmental performance appraisal, performance-based compensation, flexible job design, and participation in decision making [Lepak et al., 2006]. Strategic HRM scholars have devoted considerable effort to studying the influence of HPWS on firm performance in the past three decades and consistently found that the use of HPWS is positively related to employee and firm performance [Paauwe et al., 2013]. Indeed, recent meta-analyses have demonstrated the positive relationships between HPWS and a variety of performance outcomes [Combs et al., 2006; Jiang et al., 2012; Subramony, 2009], including employee outcomes (e.g., human capital, employee motivation), operational outcomes (e.g., productivity, service quality, and innovation), and financial outcomes (e.g., profit, return on assets, and sales growth). The purpose of this study is not to compare the results obtained from Bayesian meta-analysis to those of previous research. Instead, we use the research on HPWS as an example and focus on the relationship between

HPWS and financial performance, which is one of the most important considerations of strategic HRM research. Following the standard meta-analysis procedure, we identified 56 independent studies with the correlation data on HPWS and financial performance that were entered in the following analysis.

Before conducting Bayesian meta-analysis, we first corrected the observed correlation from each sample for unreliability by following the procedure outlined by Hunter and Schmidt (2004). Because HPWS has been considered as a formative construct (Delery, 1998) for which a high internal reliability (e.g., Cronbach's alpha) is not required, we used a reliability of 1 for the measure of HPWS. Similarly, we used a reliability of 1 for the objective measures of financial performance and used Cronbach's alpha as the reliability of the subjective measures of financial performance.

In addition, we consider firm size as a potential moderator of the relationship between HPWS and financial performance in order to test the meta-regression model of this study. Firm size is commonly included as a control variable in strategic HRM research, but its moderating effect has rarely been explored in either primary studies or a meta-analysis. Two competing hypotheses can be proposed in terms of its moderating role. On the one hand, some researchers have suggested that large organizations are likely to use more sophisticated HRM practices (e.g., HPWS) compared with small and medium enterprises [e.g., Guthrie, 2001; Jackson and Schuler, 1995]. As firm size increases, firms may also have more advantages such as economy of scale [e.g., Pfeffer and Salancik, 2003] and thus be more likely to gain benefit from their investment in HRM practices. On the other hand, large firms' financial performance may be more affected by other factors beyond human resources [Capon et al., 1990]. In this case, the role of HPWS in enhancing financial performance may be limited in large firms than in small and medium firms. Taking these considerations together, we expect that firm size may moderate the relationship between HPWS and financial performance but make no directional prediction of this effect. Firm size is usually indicated by the number of employees. Studies with average number of employees greater than 250 were coded as 1 (i.e., large firms) and the others were coded as 0 (i.e., small and

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medium firms).

Table 4 shows the summary statistics of the data used in this example. Among the total of 56 studies, 46 measured financial performance using the archival data (i.e., objective performance) and 10 used subjective measures of financial performance (i.e., subjective performance). In addition, 37 studies were coded as large firms and 19 were coded as small and medium firms. The observed correlations ranged from 0.01 to 0.52 with the sample sizes ranging from 50 to 2136.

Four power schemes are considered in the meta-analysis. First, every study is given the power coefficient of 1. In this case, every study contributes to the meta-analysis result fully and equally. This is equivalent to conduct traditional meta-analysis using Bayesian methods. Second, the reliability of financial performance of each study is used as power coefficients. The reason for this choice is that, if a measure is not reliable, only partial information will be used in meta-analysis. Third, two studies have sample sizes larger than 1000 (1212 and 2136, respectively). In order to avoid the dominant influence of the two studies on the final result, we assign them a power coefficient of 0.1 and the rest of studies a power coefficient of 1 in meta-analysis. Fourth, arguably a study with a large effect size is more likely to be published, which might cause publication bias. Therefore, reducing the influence of the studies with larger effect sizes might be helpful in reducing publication bias. In this power scheme, we set the power coefficient at 0.5 for studies with correlations larger than 0.2. For the power schemes 3 and 4, the choice of power coefficients is rather liberal. A more serious analysis might consider different levels of power coefficients.

4.1 Results of Fixed-effects Meta-analysis

We first apply the fixed-effects meta-analysis model to the example data. Table 5 shows the results using the four different power schemes. When every study is assigned the equal power coefficient of 1 (Power scheme 1), the estimated overall correlation ρ is 0.263 (ζ is the Fisher z-transformed estimate). If the reliability of financial performance is used as power coefficients

(Power scheme 2), the estimated correlation is about 0.264. However, when the two studies with the largest sample size are assigned a power coefficient of 0.5 (Power scheme 3), the estimated correlation becomes 0.226. Note the estimated correlation in this condition is significantly different from the other two correlation estimates based on the credible interval estimates. The correlations for the two studies are 0.34 and 0.45, respectively, both of which are larger than the estimated fixed-effect correlation. When no power prior is used, the two studies pull the estimates close to their correlation estimates because their larger sample sizes lead to larger weights in the estimating the overall correlation. Under the situation where the studies with larger correlations are assigned a weight 0.5 (Power scheme 4), the estimated correlation is 0.22, which is even smaller than that from Power scheme 3. This is because the larger correlations are downweighted.

4.2 Results of Random-effects Meta-analysis

Table 6 shows the results from the random-effects meta-analysis. First, the estimated correlations from the random-effects and fixed-effects methods are different (0.23 vs. 0.27) when the power priors are not used. This is because for the random-effects method, the between-study variability is considered. Therefore, extreme studies (e.g., those with unusual large sample sizes) are shrunk towards the average. Furthermore, within the random-effects method, differences in the estimated correlations are smaller. Second, only for power scheme 4, the estimated correlation shows a notable difference from the rest of the power schemes. The reason is because studies with larger correlations are downweighted. Third, in all situations, the variance estimate of τ is significant. This indicates there is sufficient variability in the studies to consider a random-effects meta-analysis to model the heterogeneity.

4.3 Results of Meta-regression

From the random-effects meta-analysis, we concluded that the population should be considered as heterogeneous.Through meta-regression analysis, we investigate whether the heterogeneity is related to firm size of different studies. Based on the results in Table 7, firm size is not significantly related to the between-study heterogeneity in the population correlations because the slope parameter β_2 is not significant regardless of the choice of power schemes. Furthermore, the results from the first three power schemes are very close. Comparing all four power schemes, power scheme 4 has a smaller intercept but a larger absolute slope. Altogether, the results do not suggest the moderating effect of the firm size on the relationship between HPWS and financial performance. It implies that HPWS used in both large firms and small and medium firms are salutary for enhancing financial performance.

5 DISCUSSION

The current study presents a Bayesian method for meta-analysis. A unique feature of our method is to enable researchers to evaluate the contribution of individual studies included in a meta-analysis through power prior. The motivation of this approach comes from the notion that not all studies should be treated equivalently when estimating the overall effect size in a meta-analysis. By developing an online program and using the example of the relationship between HPWS and financial performance, we have shown how to apply this method in practice. In the rest of this article, we briefly summarize the example results derived from the method we proposed.

In the example study, we use four power schemes to assign power coefficients to individual studies included in the meta-analysis. As shown in fixed-effects, random-effects, and meta-regression models, using the reliability of financial performance as power does not dramatically change the results obtained from regular meta-analysis that uses full information provided by each study. This is because that only ten studies used subjective measures of financial performance and the use of reliability as power would only influence how the ten out of 56 studies contribute to the final results. Moreover, the reliability for the subjective measures is typically high, so the vast majority of the information they provide still contributes to the overall effect size. If one uses another example with more subjective measures, the difference in effect size estimates between regular meta-analysis and meta-analysis using reliability as power may be more obvious. Either way, our method provides a way to evaluate whether reliability influences meta-analysis results.

When power prior is used to reduce the influence of two studies with large sample sizes, the overall effect size estimate in fixed-effects model becomes significantly different from what is obtained in the regular model, and the change is less obvious in random-effects and meta-regression models. This is because between-study variability is taken into account in random-effects models, which can shrink extreme effect sizes towards the average. However, this does not mean that using power prior to modify the impact of extremely large samples always has a larger impact on fixed-effects model than on random-effects model. It may also depend on the observed correlations of studies with large sample sizes. For example, if the correlation of a large sample is similar to the weighted average of the rest of the studies, assigning a small power to the large sample may not significantly change the overall effect size in either fixed-effects model or random-effects model.

The influence of power prior becomes more salient under power scheme 4 where studies with correlations larger than 0.2 are assigned a power coefficient of 0.5. We argue that this setting can potentially be used to deal with publication bias. For example, if we believe the studies with larger effect sizes are over-sampled, we can assign them power smaller than 1. On the other hand, if one believes the studies with smaller effect sizes are under-sampled, power coefficients larger than 1 can also be used. Certainly the choice of power prior needs careful consideration.

Bayesian meta-analysis with power prior can also be used to deal with outliers, including outliers of observed correlations and outliers of sample sizes. Traditionally, researchers often eliminate the most extreme data points to attenuate the influence of outliers on overall effect size estimation [e.g., Hedges, 1992; Huber, 1981; Tukey, 1960]. This is similar to assigning a power coefficient of 0 to studies considered as outliers and using no information of the eliminated studies in analysis. However, rather than deleting the data points completely, researchers can also choose to use only a small part of their information by assigning a small non-zero power

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coefficient to those studies.

One important issue that is out of the discussion of this article is what power coefficient should be assigned to each study in meta-analysis with power prior. The method proposed in this study cannot determine whether a power prior scheme is realistic or not to reflect the contribution of each study to the final results. It is more reasonable for researchers who are familiar with the nature of the included studies to make the decisions. The general guideline is to identify the criteria that can indicate the credibility of research findings and use it to guide power prior decision in meta-analysis. One attempt of this study is to use reliability as power coefficients for studies relying on subjective measures, which may reduce the over-correction for unreliability due to extremely low reliability. In addition, we recommend that one should always compare the results from the analysis with and without power priors to inform the influence of the use of power priors. We encourage more efforts to further explore this issue in the future.

This study can be improved and extended in many ways. First, in both random-effects meta-analysis and meta-regression, we assume that the random effects follow a normal distribution. This assumption might not be valid when there are extreme values. Further study can incorporate robust Bayesian analysis to deal with the problem [e.g., Zhang et al., 2013]. Second, the current study has focused on the development of the method for correlation. However, the method can be applied to other effect sizes such as mean differences and odds ratios. Third, Ibrahim and Chen [2000b] has suggested that the power coefficients in the power prior can be estimated by specifying a distribution for the power coefficient. In the literature, a beta distribution has been used. A future study can investigate this in meta-analysis.

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References

Brannick, M. T. (2001). Implications of empirical bayes meta-analysis for test validation. *Journal of Applied Psychology*, 86:468–480.

Capon, N., Farley, J. U., and Hoenig, S. (1990). Determinants of financial performance: a meta-analysis. *Management Science*, 36(10):1143–1159.

Carlin, B. P. and Louis, T. A. (1996). *Bayes and empirical Bayes methods for data analysis*. Chapman & Hall.

Carlin, J. B. (1992). Meta-analysis for 2× 2 tables: A bayesian approach. *Statistics in medicine*, 11(2):141–158.

Combs, J., Liu, Y., Hall, A., and Ketchen, D. (2006). How much do high-performance work practices matter? a meta-analysis of their effects on organizational performance. *Personnel Psychology*, 59(3):501–528.

Crook, T. R., Todd, S. Y., Combs, J. G., Woehr, D. J., and Ketchen Jr, D. J. (2011). Does human capital matter? a meta-analysis of the relationship between human capital and firm performance. *Journal of applied psychology*, 96(3):443.

Field, A. P. (2001). Meta-analysis of correlation coefficients: A monte carlo comparison of fixed- and random-effects methods. *Psychological Methods*, 6:161–180.

Fisher, R. A. et al. (1921). On the" probable error" of a coefficient of correlation deduced from a small sample. *Metron*, 1:3–32.

Gelman, A., Carlin, J. B., Stern, H. S., and Rubin, D. B. (2003). *Bayesian data analysis*. Chapman & Hall/CRC.

Gill, J. (2002). *Bayesian Methods: A Social and Behavioral Sciences Approach*. Chapman & Hall/CRC.

Greenland, S. (2000). Principles of multilevel modelling. *International Journal of Epidemiology*, 29(1):158–167.

Guthrie, J. P. (2001). High-involvement work practices, turnover, and productivity: Evidence from new zealand. *Academy of management Journal*, 44(1):180–190.

Hedges, L. V. (1992). Meta-analysis. *Journal of Educational and Behavioral Statistics*, 17(4):279–296.

Hedges, L. V. and Olkin, I. (1985). *Statistical methods for meta-analysis*. Academic, New York.

Hedges, L. V. and Vevea, J. L. (1998). Fixed- and random-effects models in meta-analysis. *Psychological methods*, 3(4):486–504.

Huber, P. J. (1981). *Robust statistics*. John Wiley & Sons, Inc, New York.

Hunter, J. E. and Schmidt, F. L. (2004). *Methods of meta-analysis: Correcting error and bias in research findings*. Sage, Newbury Park, CA, 2nd edition.

Ibrahim, J. G. and Chen, M. H. (2000a). Power prior distributions for regression models. *Statistical Science*, 15:46–60.

Ibrahim, J. G. and Chen, M.-H. (2000b). Power prior distributions for regression models. *Statistical Science*, pages 46–60.

Jackson, S. E. and Schuler, R. S. (1995). Understanding human resource management in the context of organizations and their environments. *Strategic Human Resource Management*, 46:237–264.

Jeffreys, H. (1946). An invariant form for the prior probability in estimation problems. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 186(1007):453–461.

Jiang, K., Lepak, D. P., Hu, J., and Baer, J. C. (2012). How does human resource management influence organizational outcomes? a meta-analytic investigation of mediating mechanisms. *Academy of management Journal*, 55(6):1264–1294.

Lepak, D. P., Liao, H., Chung, Y., and Harden, E. E. (2006). A conceptual review of human resource management systems in strategic human resource management research. *Research in personnel and human resources management*, 25(1):217–271.

Morris, C. and Normand, S. (1992). Hierarchical models for combining information and for meta-analyses. *Bayesian statistics*, 4:321–344.

Paauwe, J., Wright, P., and Guest, D. (2013). Hrm and performance: what do we know and where should we go? In Paauwe, J., Guest, D. E., and Wright, P. M., editors, *HRM & performance: Achievements & challenges*, pages 1–14. Wiley.

Pfeffer, J. and Salancik, G. R. (2003). *The external control of organizations: A resource dependence perspective*. Stanford University Press.

Podsakoff, P. M., MacKenzie, S. B., Lee, J.-Y., and Podsakoff, N. P. (2003). Common method biases in behavioral research: a critical review of the literature and recommended remedies. *Journal of applied psychology*, 88(5):879.

Rosenthal, R. (1991). *Meta-analytic procedures for social research*, volume 6. Sage.

Schmidt, F. L. and Hunter, J. E. (1977). Development of a general solution to the problem validity generalization. *Journal of Applied Psychology*, 62:529–540.

Schmidt, F. L., Oh, I.-S., and Hayes, T. L. (2009). Fixed-versus random-effects models in meta-analysis: Model properties and an empirical comparison of differences in results. *British Journal of Mathematical and Statistical Psychology*, 62(1):97–128.

Schmidt, F. L. and Raju, N. S. (2007). Updating meta-analytic research findings: Bayesian approaches versus the medical model. *Journal of Applied Psychology*, 92(2):297.

Smith, T. C., Spiegelhalter, D. J., and Thomas, A. (1995). Bayesian approaches to random-effects meta-analysis: a comparative study. *Statistics in medicine*, 14(24):2685–2699.

Steel, P. D. G. and Kammeyer-Mueller, J. (2008). Bayesian variance estimation for meta-analysis: Quantifying our uncertainty. *Organizational Research Methods*, 11(1):54–78.

Strenio, J. F., Weisberg, H. I., and Bryk, A. S. (1983). Empirical bayes estimation of individual growth-curve parameters and their relationship to covariates. *Biometrics*, pages 71–86.

Subramony, M. (2009). A meta-analytic investigation of the relationship between hrm bundles and firm performance. *Human resource management*, 48(5):745–768.

Sutton, A. J. and Abrams, K. R. (2001). Bayesian methods in meta-analysis and evidence synthesis. *Statistical Methods in Medical Research*, 10(4):277–303.

Tukey, J. W. (1960). A survey of sampling from contaminated distributions. In Olkin, I., Ghurye, J., Hoeffding, W., Madoo, W. G., and Mann, H., editors, *Contributions to probability and statistics*. Stanford University Press, Stanford, CA.

Zhang, Z., Lai, K., Lu, Z., and Tong, X. (2013). Bayesian inference and application of robust growth curve models using student’s t distribution. *Structural equation modeling*, 20:47–78.

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6 APPENDIX A

With the prior and the information from the first study, the posterior, based on Bayes' Theorem, is

$$
\begin{aligned}
p(\zeta|z_1) &= \frac{p(\zeta)p(z_1|\zeta)}{p(z_1)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_0}}\exp\left[-\frac{(\zeta-\zeta_0)^2}{2\psi_0}\right]\frac{1}{\sqrt{2\pi\phi_1}}\exp\left[-\frac{(z_1-\zeta)^2}{2\phi_1}\right]}{p(z_1)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_0}}\frac{1}{\sqrt{2\pi\phi_1}}\exp\left[-(\frac{1}{2\psi_0}+\frac{1}{2\phi_1})\zeta^2+2(\frac{1}{2\psi_0}\zeta_0+\frac{1}{2\phi_1}z_1)\zeta-(\frac{\zeta_0^2}{2\psi_0}+\frac{z_1^2}{2\phi_1})\right]}{p(z_1)}, \\
&= \frac{D\exp\left[-\frac{1}{2}(A\zeta^2+2B\zeta+C)\right]}{p(z_1)} \\
&= \frac{D\exp\left[-\frac{(\zeta-\frac{B}{A})^2}{2\frac{1}{A}}-\frac{1}{2}(C-\frac{B^2}{A})\right]}{p(z_1)}
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \frac{1}{\psi_0}+\frac{1}{\phi_1} \\
B &= \frac{1}{\psi_0}\zeta_0+\frac{1}{\phi_1}z_1. \\
C &= \frac{\zeta_0^2}{\psi_0}+\frac{z_1^2}{\phi_1}
\end{aligned}
$$

The denominator is

$$
\begin{aligned}
p(z_1) &= \int_{-\infty}^{+\infty}(D\exp\left[-\frac{(\zeta-\frac{B}{A})^2}{2\frac{1}{A}}-\frac{1}{2}(C-\frac{B^2}{A})\right])d\zeta \\
&= D\exp\left[-\frac{1}{2}(C-\frac{B^2}{A})\right]\times\sqrt{2\pi\frac{1}{A}}
\end{aligned}
$$

Therefore, the posterior is

$$
p(\zeta|z_1) = \frac{1}{\sqrt{2\pi\frac{1}{A}}}\exp\left[-\frac{(\zeta-\frac{B}{A})^2}{2\frac{1}{A}}\right],
$$

a normal distribution with mean

$$
B/A = \frac{\frac{1}{\psi_0}\zeta_0+\frac{1}{\phi_1}z_1}{\frac{1}{\psi_0}+\frac{1}{\phi_1}} = \frac{\phi_1\zeta_0+\psi_0 z_1}{\phi_1+\psi_0}
$$

and variance

$$1/A = \frac{1}{\frac{1}{\psi_0} + \frac{1}{\phi_1}}.$$

7 APPENDIX B

With the power parameter α_1, the posterior

$$\begin{aligned}
p(\zeta|z_1) &= \frac{p(\zeta)[p(z_1|\zeta)]^{\alpha_1}}{p(z_1)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_0}} \exp\left[-\frac{(\zeta-\zeta_0)^2}{2\psi_0}\right] \left\{\frac{1}{\sqrt{2\pi\phi_1}} \exp\left[-\frac{(z_1-\zeta)^2}{2\phi_1}\right]\right\}^{\alpha_1}}{p(z_1)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_0}} \left(\frac{1}{\sqrt{2\pi\phi_1}}\right)^{\alpha_1} \exp\left[-\frac{(\zeta-\zeta_0)^2}{2\psi_0} - \exp\left[-\frac{(z_1-\zeta)^2}{2\phi_1/\alpha_1}\right]\right]}{} \\
&= \frac{D \exp\left[-(\frac{1}{2\psi_0} + \frac{1}{2\phi_1^*})\zeta^2 + 2(\frac{1}{2\psi_0}\zeta_0 + \frac{1}{2\phi_1^*}z_1)\zeta - (\frac{\zeta_0^2}{2\psi_0} + \frac{z_1^2}{2\phi_1^*})\right]}{p(z_1)}, \\
&= \frac{D \exp\left[-\frac{1}{2}(A\zeta^2 + 2B\zeta - C)\right]}{p(z_1)} \\
&= \frac{D \exp\left[-\frac{(\zeta-\frac{B}{A})^2}{2\frac{1}{A}} - \frac{1}{2}(C - \frac{B^2}{A})\right]}{p(z_1)}
\end{aligned}$$

where

$$\begin{aligned}
A &= \frac{1}{\psi_0} + \frac{1}{\phi_1^*} \\
B &= \frac{1}{\psi_0}\zeta_0 + \frac{1}{\phi_1^*}z_1, \\
C &= \frac{\zeta_0^2}{\psi_0} + \frac{z_1^2}{\phi_1^*}
\end{aligned}$$

and $\phi_1^* = \phi_1/\alpha_1$. From Appendix A, the posterior is $N(B/A, 1/A)$ where

$$\begin{aligned}
B/A &= \frac{\frac{1}{\psi_0}\zeta_0 + \frac{1}{\phi_1^*}z_1}{\frac{1}{\psi_0} + \frac{1}{\phi_1^*}} = \frac{\phi_1^*\zeta_0 + \psi_0 z_1}{\phi_1^* + \psi_0} = \frac{\frac{\phi_1}{\alpha_1}\phi_1 + \psi_0 z_1}{\frac{\phi_1}{\alpha_1} + \psi_0} \\
1/A &= \frac{1}{\frac{1}{\psi_0} + \frac{1}{\phi_1^*}} = \frac{1}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1}}.
\end{aligned}$$

8 APPENDIX C

Show the posterior

$$
\begin{aligned}
p(\zeta|z_1, z_2, \alpha_1, \alpha_2) &= \frac{p(\zeta|z_1, \alpha_1)[p(z_2|\zeta)]^{\alpha_2}}{p(z_2)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_1^*}} \exp\left[-\frac{(\zeta-\zeta_1^*)^2}{2\psi_1^*}\right] \left\{\frac{1}{\sqrt{2\pi\phi_2}} \exp\left[-\frac{(z_2-\zeta)^2}{2\phi_2}\right]\right\}^{\alpha_2}}{p(z_2)} \\
&= \frac{\frac{1}{\sqrt{2\pi\psi_1^*}} \left(\frac{1}{\sqrt{2\pi\phi_2}}\right)^{\alpha_2} \exp\left[-\frac{(\zeta-\zeta_1^*)^2}{2\psi_1^*} - \exp\left[-\frac{(z_1-\zeta)^2}{2\phi_1/\alpha_1}\right]\right]}{p(z_2)} \\
&= \frac{D \exp\left[-\frac{1}{2}(A\zeta^2 + 2B\zeta - C)\right]}{p(z_1)} \\
&= \frac{D \exp\left[-\frac{(\zeta-\frac{B}{A})^2}{2\frac{1}{A}} - \frac{1}{2}(C - \frac{B^2}{A})\right]}{p(z_1)}
\end{aligned}
$$

The denominator is

$$
\begin{aligned}
p(z_1) &= \int_{-\infty}^{+\infty} (D \exp\left[-\frac{(\zeta - \frac{B}{A})^2}{2\frac{1}{A}} - \frac{1}{2}(C - \frac{B^2}{A})\right]) d\zeta \\
&= D \exp\left[-\frac{1}{2}(C - \frac{B^2}{A})\right] \times \sqrt{2\pi\frac{1}{A}}
\end{aligned}
$$

The posterior is $N(B/A, 1/A)$ where

$$
\begin{aligned}
B/A &= \frac{\frac{1}{\psi_1^*}\zeta_1^* + \frac{1}{\phi_2^*}z_2}{\frac{1}{\psi_1^*} + \frac{1}{\phi_2^*}} = \frac{\frac{1}{\psi_0}\zeta_0 + \frac{\alpha_1}{\phi_1}z_1 + \frac{\alpha_2}{\phi_2}z_2}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1} + \frac{\alpha_2}{\phi_2}} \\
1/A &= \frac{1}{\frac{1}{\psi_1^*} + \frac{1}{\phi_1^*}} = \frac{1}{\frac{1}{\psi_0} + \frac{\alpha_1}{\phi_1} + \frac{\alpha_2}{\phi_2}}
\end{aligned}
$$

9 APPENDIX D

The joint posterior distribution is

$$
\begin{aligned}
p(\zeta, \tau, \zeta_i|z_i, \phi_i, \alpha_i) &\propto p(\zeta)p(\tau)\prod_{i=1}^{m} p_{\alpha_i}(z_i, \zeta_i|\zeta, \tau) \\
&= p(\zeta)p(\tau)\prod_{i=1}^{m} [p_{\alpha_i}(z_i|\zeta_i, \phi_i)p(\zeta_i|\zeta, \tau)]
\end{aligned}
$$

$$\propto \frac{1}{\sqrt{2\pi\psi_0}} \exp\left[-\frac{(\zeta-\zeta_0)^2}{2\psi_0}\right] \tau^{-\delta_0-1} \exp\left[-\frac{\gamma_0}{\tau}\right]$$
$$\times \left[\prod_{i=1}^{m} (2\pi\phi_i)^{-\alpha_i/2}\right] \exp\left[-\sum_{i=1}^{m} \frac{(z_i-\zeta_i)^2}{2\phi_i/\alpha_i}\right] (2\pi\tau)^{-m/2} \exp\left[-\frac{\sum_{i=1}^{m}(\zeta_i-\zeta)^2}{2\tau}\right].$$

Now we obtain the conditional posterior distributions.

First, we get the conditional posterior distribution of τ, which is

$$p(\tau|\zeta_i, \zeta, \alpha_i) \propto \tau^{-\delta_0-1-m/2} \exp\left[-\frac{2\gamma_0 + \sum(\zeta_i-\zeta)^2}{2\tau}\right].$$

Therefore, the posterior is inverse Gamma distribution $\mathrm{IG}(\delta_0 + m/2, \gamma_0 + [\sum(\zeta_i-\zeta)^2]/2)$.

Second, the conditional posterior distribution of ζ is

$$p(\zeta|\zeta_i, \tau) \propto \exp\left[-\frac{(\zeta-\zeta_0)^2}{2\psi_0} - \frac{\sum(\zeta_i-\zeta)^2}{2\tau}\right].$$

Therefore, the conditional posterior is a normal distribution

$$N\left(\frac{\frac{\sum_{i=1}^{m}\zeta_i}{\tau} + \frac{\zeta_0}{\psi_0}}{\frac{m}{\tau} + \frac{1}{\psi_0}}, \frac{1}{\frac{m}{\tau} + \frac{1}{\psi_0}}\right).$$

Third, the conditional posterior distribution of ζ_i is

$$p(\zeta_i|\zeta, z_i, \tau, \alpha_i) \propto \exp\left[-\frac{(z_i-\zeta_i)^2}{2\phi_i/\alpha_i} - \frac{(\zeta_i-\zeta)^2}{2\tau}\right],$$

which is a normal distribution

$$N\left(\frac{\frac{\zeta}{\tau} + \frac{z_i\alpha_i}{\phi_i}}{\frac{1}{\tau} + \frac{\alpha_i}{\phi_i}}, \frac{1}{\frac{1}{\tau} + \frac{\alpha_i}{\phi_i}}\right).$$

10 APPENDIX E

The joint posterior distribution for the meta-regression model is

$$\begin{aligned}
p(\boldsymbol{\beta}, \tau|z_i, \zeta_i, \alpha_i) &\propto p(\boldsymbol{\beta})p(\tau)\prod p_{\alpha_i}(z_i, \zeta_i|\boldsymbol{\beta}, \tau) \\
&= p(\boldsymbol{\beta})p(\tau)\prod p_{\alpha_i}(z_i|\zeta_i, \phi_i)p(\zeta_i|\boldsymbol{\beta}, \tau) \\
&\propto |\boldsymbol{\Psi}_0|^{-1/2} \exp\left[-\frac{1}{2}(\boldsymbol{\beta}-\boldsymbol{\zeta}_0)'\boldsymbol{\Psi}_\mathbf{0}^{-1}(\boldsymbol{\beta}-\boldsymbol{\zeta}_0)\right] \tau^{-\delta_0-1} \exp\left[-\frac{\gamma_0}{\tau}\right] \\
&\quad \times \prod \left\{(2\pi\phi_i)^{-\alpha_i/2} \exp\left[-\sum \frac{(z_i-\zeta_i)^2}{2\phi_i/\alpha_i}\right] (2\pi\tau)^{-m/2} \exp\left[-\frac{\sum(\zeta_i - \mathbf{x}_i\boldsymbol{\beta})^2}{2\tau}\right]\right\}.
\end{aligned}$$

The conditional posterior distribution of τ is

$$p(\tau|\boldsymbol{\beta},\zeta_i) \propto \tau^{-(\delta_0+m/2)-1} \exp\left[-\frac{\gamma_0 + \sum(\zeta_i - \mathbf{x}_i\boldsymbol{\beta})^2/2}{\tau}\right] \tag{6}$$

Therefore, $\tau|\boldsymbol{\beta},\zeta_i \sim IG(\delta_0 + m/2, \gamma_0 + \sum(\zeta_i - \mathbf{x}_i\boldsymbol{\beta})^2/2)$. The conditional posterior distribution for $\boldsymbol{\beta}$ is

$$p(\boldsymbol{\beta}|\tau,\zeta_i) \propto \exp\left[-\frac{1}{2}(\boldsymbol{\beta}-\boldsymbol{\zeta}_0)'\boldsymbol{\Psi}_0^{-1}(\boldsymbol{\beta}-\boldsymbol{\zeta}_0)\right]\exp\left[\frac{\sum(\zeta_i - \mathbf{x}_i\boldsymbol{\beta})^2}{2\tau}\right]. \tag{7}$$

Let $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \cdots, \zeta_m)'$ be the vector of $\zeta_i's$, and $\hat{\beta}$ be the least square estimate such that $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\zeta}$ with $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_m)'$ as the design matrix. Then the conditional posterior distribution of β is a multivariate normal distribution

$$N\left(\left(\boldsymbol{\Psi}_0^{-1} + \frac{\mathbf{X}'\mathbf{X}}{\tau}\right)^{-1}\left(\boldsymbol{\Psi}_0^{-1}\boldsymbol{\zeta}_0 + \frac{\mathbf{X}'\mathbf{X}}{\tau}\hat{\boldsymbol{\beta}}\right), (\boldsymbol{\Psi}_0^{-1} + \frac{\mathbf{X}'\mathbf{X}}{\tau})^{-1}\right)$$

For ζ_i, its conditional distribution is

$$p(\zeta_i|\beta,\tau) \propto \exp\left[-\frac{(z_i-\zeta_i)^2}{2\phi_i/\alpha_i}\right]\exp\left[-\frac{(\zeta_i - X_i\beta)^2}{2\tau}\right],$$

a normal distribution

$$N(\frac{\frac{\alpha_i z_i}{\phi_i} + \frac{X_i\beta}{\tau}}{\frac{\alpha_i}{\phi_i} + \frac{1}{\tau}}, \frac{1}{\frac{\alpha_i}{\phi_i} + \frac{1}{\tau}}).$$

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Table 1: The influence of the selection of power parameters for a single study

Data	z-transformation	Variance estimate
$r_1 = 0.5$	0.549	0.04
Prior	0	1
Power schemes	Posterior	
α_1	Mean	Variance
0	0	1
0.1	0.392	0.286
0.2	0.458	0.167
0.3	0.485	0.118
0.4	0.499	0.091
0.5	0.509	0.074
0.6	0.515	0.063
0.7	0.520	0.054
0.8	0.523	0.048
0.9	0.526	0.043
1	0.528	0.038

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Table 2: The influence of the selection of power parameters for combining two studies

Data		z-transformation	Variance estimate
$r_1 = 0.5$		0.549	0.04
$r_2 = 0$		0	.01
Prior		0	100
Power schemes		Posterior	
α_1	α_2	Mean	Variance
0	0	0	100
1	0	0.549	0.040
0	1	0.000	0.010
0.1	1	0.013	0.010
1	0.1	0.392	0.029
0.5	0.5	0.110	0.016
0.2	1	0.026	0.010
1	0.2	0.305	0.022
0.2	0.8	0.032	0.012
0.8	0.2	0.275	0.025
1	1	0.110	0.008

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Table 3: The influence of the use of power parameters on random-effects meta-analysis

Data			z-transformation		Variance estimate	
$r_1 = 0.5$			0.549		0.01	
$r_2 = 0$			0		0.04	
$r_3 = -0.5$			-0.549		0.01	
Prior			0		100	
Power scheme			Posterior mean			
α_1	α_2	α_3	ρ	ρ_1	ρ_2	ρ_3
1	1	1	-0.002	0.482	-0.001	-0.482
1	1	0.1	0.061	0.476	0.022	-0.305
1	1	0.01	0.215	0.469	0.099	0.099

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Table 4: Summary statistics

	Minimum	Mean	Median	Maximum	Standard deviation
Correlation	0.01	0.22	0.200	0.52	0.13
Sample size	50	281	191	2136	325
Reliability	0.74	0.97	1	1	0.07
	Small & Medium: 19			Large: 37	
	Objective studies: 46			Subjective studies: 10	

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Table 5: Results from fixed-effects meta-analysis

Power Scheme		Estimate	sd	CI		DIC
1	ζ	0.27*	0.008	0.254	0.285	184
	ρ	0.26*	0.007	0.249	0.278	
2	ζ	0.27*	0.008	0.255	0.286	175.9
	ρ	0.26*	0.008	0.249	0.279	
3	ζ	0.23*	0.009	0.212	0.247	72.11
	ρ	0.23*	0.008	0.209	0.242	
4	ζ	0.22*	0.009	0.205	0.242	77.44
	ρ	0.22*	0.009	0.202	0.237	

Note. * $p < 0.05$. Power scheme 1: each study is given a power coefficient of 1. Power scheme 2: the reliability of financial performance is used as power coefficients. Power scheme 3: the two studies with the largest sample sizes are given a power coefficients of 0.1. Power scheme 4: studies with correlations larger than 0.2 are given a power coefficient of 0.5, otherwise, 1.

Table 6: Results from random-effects meta-analysis

Power Scheme		Estimate	sd	CI		DIC
1	ζ	0.23*	0.02	0.191	0.269	-98.45
	τ	0.016*	0.004	0.01	0.026	
	ρ	0.226*	0.019	0.189	0.263	
2	ζ	0.23*	0.02	0.191	0.27	-97.18
	τ	0.016*	0.004	0.01	0.026	
	ρ	0.226*	0.019	0.189	0.263	
3	ζ	0.228*	0.02	0.19	0.267	-93.99
	τ	0.016*	0.004	0.009	0.025	
	ρ	0.224*	0.019	0.187	0.261	
4	ζ	0.218*	0.02	0.178	0.259	-85.4
	τ	0.015*	0.004	0.008	0.024	
	ρ	0.214*	0.019	0.177	0.253	

Note. * $p < 0.05$. Power scheme 1: each study is given a power coefficient of 1. Power scheme 2: the reliability of financial performance is used as power coefficients. Power scheme 3: the two studies with the largest sample sizes are given a power coefficients of 0.1. Power scheme 4: studies with correlations larger than 0.2 are given a power coefficient of 0.5, otherwise, 1.

Table 7: Results from meta-regression[2,3]

Power Scheme		Estimate	sd	CI		DIC
1	β_1(intercept)	0.248*	0.034	0.181	0.316	-97.9
	β_2(size)	-0.028	0.042	-0.113	0.053	
	τ	0.017*	0.004	0.01	0.026	
2	β_1(intercept)	0.249*	0.034	0.181	0.317	-96.63
	β_2(size)	-0.029	0.042	-0.113	0.053	
	τ	0.016*	0.004	0.01	0.026	
3	β_1(intercept)	0.245*	0.034	0.179	0.312	-93.5
	β_2(size)	-0.027	0.042	-0.111	0.054	
	τ	0.016*	0.004	0.009	0.025	
4	β_1(intercept)	0.24*	0.035	0.172	0.31	-84.79
	β_2(size)	-0.034	0.043	-0.121	0.048	
	τ	0.015*	0.004	0.008	0.025	

Note. * $p < 0.05$. Power scheme 1: each study is given a power coefficient of 1. Power scheme 2: the reliability of financial performance is used as power coefficients. Power scheme 3: the two studies with the largest sample sizes are given a power coefficients of 0.1. Power scheme 4: studies with correlations larger than 0.2 are given a power coefficient of 0.5, otherwise, 1.

Figure 1: The interface of the online software metacorr